

# ANALYSIS OF HTS FILTERS USING NOVEL NONLINEAR PHENOMENOLOGICAL TWO FLUID MODEL:

Mohamed Megahed, Samir El-Ghazaly and Aly Fathy\*

Telecommunications Research Center  
Arizona State University, Tempe, AZ 85287-5706

\*David Sarnoff Research Center  
Princeton, NJ 08543-5300

## ABSTRACT

A novel nonlinear phenomenological two fluid model for High Temperature Superconductor (HTS) is proposed. The model combines the physics associated with the Ginzburg-Landau expressions and the required simplicity obtained from the linear London's equations. An empirical formula for the nonlinear surface impedance is suggested based on the developed model. Analysis of HTS filter using this novel nonlinear formulation is presented.

## INTRODUCTION

HTS filters have potential application in base band communication systems. However, the material exhibits severe nonlinearity, which degrades the filter response. Modeling the nonlinearity in HTS requires a simple formulation, that can be easily implemented in CAD environment. However, the two-fluid and London models did not consider the bidirectional coupling between the thermodynamics and electrodynamics in a superconducting system. In this paper, a new phenomenological nonlinear two-fluid model for HTS is proposed. Also, an empirical formula, based on the model, for the nonlinear surface resistance is developed. The model was used to study the effects for the nonlinearity on superconducting filters.

## NONLINEAR PHENOMENOLOGICAL TWO FLUID MODEL

The main macroscopic parameters of superconductors are the magnetic field penetration depth and the normal conductivity. Ginzburg-Landau (GL) theory results in a spatial, field and temperature dependent macroscopic parameters. The magnetic field penetration depth can be calculated from the following expression [1]:

$$\lambda_s(H(T), T) = \frac{\lambda_s(0,0)}{\sqrt{\left(\frac{\psi(H(T), T)}{\psi(0, T)}\right)^\alpha * \left(1 - \left(\frac{T}{T_c}\right)^\beta\right)}} \quad (1)$$

where  $\lambda_s(0,0)$  is the low field penetration depth measured at  $T=0$  and  $H=0$ , and  $T_c$  is the critical temperature for the superconductor calculated at  $H=0$ . The parameters  $\alpha$  and  $\beta$  may be obtained from experimental studies.

It is known that fraction of the conduction electrons in the superfluid state  $n_s$  could be assumed to vary from unity at  $H=0$  to zero at the field of the transition to the completely normal state  $H_c$ . This postulate is valid at temperature  $T=0$ . The critical magnetic field  $H_c$  is computed at  $T=0$ . This argument is expected to lead to dependence similar to the Corter-Casimir thermal relation. The fraction of the superfluid electrons can be deduced from the following expression:

$$\frac{n_s}{n_{s\infty}} = 1 - \left(\frac{H}{H_c}\right)^\alpha \quad (2)$$

The parameter  $a$  may be computed from experimental studies. A temperature and field dependence formulation can be obtained by combining Eq. 2 with Gorter-Casimir temperature relation,

$$\frac{n_s}{n_{s\infty}} = 1 - \left(\frac{T}{T_c}\right)^4 \quad (3)$$

and by using the result obtained from GL theory Eq. 1, the fraction of the conduction electrons in the superfluid state  $n_s$  can be written as

$$\frac{n_s(H,T)}{n_{s\infty}} = \left[1 - \left(\frac{H}{H_c(T)}\right)^\alpha\right] \left[1 - \left(\frac{T}{T_c}\right)^\beta\right] \quad (4)$$

and

$$H_c(T) \approx H_{co} \left[1 - \left(\frac{T}{T_c}\right)^2\right] \quad T \leq T_c \quad (5)$$

where  $H_{co}$  is the thermodynamic critical field at zero temperature. Actually,  $H_c(T)$  will be assumed as a given experimental quantity.

Using the conservation of electrons law, the fraction of the normalfluid electrons can be deduced from the following expression

$$\frac{n_n(H,T)}{n} = 1 - \frac{n_s(H,T)}{n_{s\infty}} \quad (6)$$

which can be rewritten in terms of temperature and field dependences as

$$\frac{n_n(H,T)}{n} = \left\{1 - \left[1 - \left(\frac{H}{H_c(T)}\right)^\alpha\right] \left[1 - \left(\frac{T}{T_c}\right)^\beta\right]\right\} \quad (7)$$

## MACROSCOPIC MODEL OF NONLINEAR CONSTITUTIVE RELATIONS IN HTS

The spatial, field and temperature dependent magnetic field penetration depth  $\lambda_s$  for the

superconductor can be calculated from London expressions as:

$$\lambda_s(H(T),T) = \frac{\lambda_{s(0,0)}}{\sqrt{(1-h^\alpha)(1-t^\beta)}} \quad (8)$$

where  $\lambda_{s(0,0)}$  is the low field penetration depth measured at  $T = 0$  and  $H = 0$ ,  $h = H(\bar{r})/H_c(T)$ ,  $t = T/T_c$ , and  $T_c$  is the critical temperature for the superconductor calculated at  $H = 0$ . The parameters  $\alpha$  and  $\beta$  may be obtained from experimental studies.

The normalfluid current density is deduced from Ohm's law, and the corresponding normal conductivity is expressed as follows :

$$\sigma_n(H(\bar{r}),T) = \sigma_n(H_c/T_c) \left[1 - (1-h^\alpha)(1-t^\beta)\right] \quad (9)$$

where  $\sigma_n(H_c/T_c)$  is the maximum normal conductivity measured either at  $T = T_c$  or  $H = H_c$  and  $H_c$  is the critical magnetic field for the superconductor calculated at  $T = 0$ . It is obvious that the described model combines the physics associated with the GL phenomenological model and the required simplicity obtained from the linear London's model.

## HTS NONLINEAR SURFACE IMPEDANCE

The proposed nonlinear two-fluid model is verified using the experimental results obtained for the normalized surface impedance. An empirical formula proposed by Pippard that agrees very closely with experimental measurements for conventional superconductor [2] gives

$$\Re_s(h,t,\omega) \equiv \omega^2 \frac{t^4(1-t^2)}{(1-t^4)^2} \quad (10)$$

However, this expression fails to predict the temperature dependence for the surface resistance of the YBCO HTS. The formula that fits mostly with the experimental results

depicted for the temperature and field dependence of the YBCO HTS, as will be shown later, can be expressed as follows

$$\Re_s(h, t, \omega) \equiv \omega^2 \frac{[1 - (1 - h^\alpha)(1 - t^\beta)]}{[(1 - h^\alpha)(1 - t^\beta)]^{1/2}} \quad (11)$$

where  $\alpha$  and  $\beta$  are obtained empirically. Their values depend on the operating region of interest.

## APPLICATION ON HTS FILTER

The presented low-pass filter is the one published in [3], and shown in Fig. 1. The filter is chosen just to demonstrate the effect of the nonlinearity associated with HTS on filters in general. The long rectangular patch is 2.54x20.32 mm. The strip widths of the input and output ports are 2.413 mm. The HTS strip thickness is chosen to equal 0.4  $\mu\text{m}$ . The macroscopic parameters of the YBCO films are as follows: low field penetration depth  $\lambda_s(0) = 1670 \text{ \AA}$  and normal conductivity  $\sigma_n(T_c) = 1.6 \times 10^6 \text{ S/m}$ . The critical temperature  $T_c$  for the YBCO equals to 86.4K. The analysis is performed using the full-wave finite-difference time-domain. The algorithm is described in details elsewhere [1].

## RESULTS AND DISCUSSION

The surface resistance of  $\text{YBa}_2\text{Cu}_3\text{O}_{7-x}$  HTS film with critical temperature of 86.4 K, and magnetic field penetration depth  $\lambda_L(0)$  equals 0.167  $\mu\text{m}$  is estimated using the proposed nonlinear two fluid model. The calculated results are compared with the measured data in [3]. Fig. 2 shows the critical magnetic field, calculated using Eq. (4), as a function of the normalized temperature. Although, this formula is assumed for conventional superconductor, it fits well the measured data for the YBCO in this operation region.

The temperature dependence for the surface resistance at zero applied magnetic field is calculated using Eq. (11), with  $\beta = 3/2$ . The operating frequency equals to 1.5 GHz. The results are compared with the measured data

presented in [4], and shown in Fig. 3. It is seen that fair agreement is obtained between  $t = 0.6$  and  $t = 0.95$ . This operating region includes the liquid nitrogen boiling temperature, 77 K, where all the new HTS material operates.

The surface resistance as function of both the temperature and magnetic field at 1.5 GHz is depicted in Fig. 4. The surface resistance is calculated using Eq. (11), with  $\alpha = 3/4$  and  $\beta = 3/2$ . A comparison between the calculated results and the experimental data is conducted and presented in Fig. 4. A good agreement between the measured and the calculated data in the practical operation region of the YBCO HTS can be observed. The presented results show that the effect of the magnetic field on the HTS material is more pronounced than the temperature effect.

The insertion loss of the low pass filter is shown in Fig. 5. The scattering parameters is obtained for different values of the applied power. The applied magnetic field is normalized to the critical magnetic field value. The results are generated for the linear HTS using London's model as well as the nonlinear HTS using the proposed nonlinear model. It is clear that as the applied power increases the filter performance deteriorates. The effect on the trailing edge of the S21 characteristics is more pronounced. As the applied field increases, the stopband frequency is also shifted due to the nonlinearity of the HTS.

## CONCLUSION

A novel nonlinear phenomenological two-fluid model is proposed for superconducting materials. A macroscopic model for the nonlinear constitutive relations is also suggested. An empirical formula for the surface impedance of HTS that agrees very closely with experimental measurements for YBCO superconductors is developed. These compact models are validated and verified by comparing the calculated results with data obtained from experimental measurements. This model combines the physics associated with the GL phenomenological model and the required simplicity obtained from the linear London's model. The nonlinear model is successfully applied to study the effect of the nonlinearity on HTS filters.

## REFERENCES

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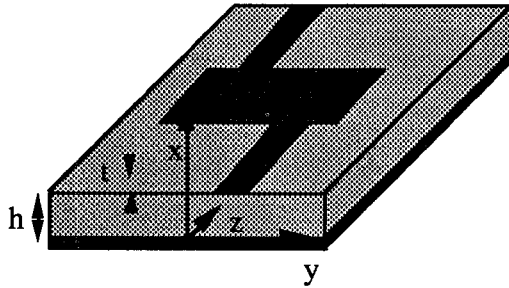


Fig. 1 Low pass filter structure

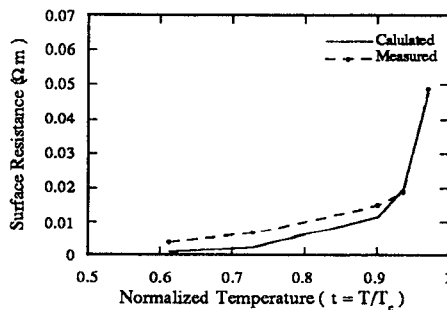


Fig. 2 Comparison between the calculated and measured [4] critical magnetic field for YBCO HTS as function of temperature.

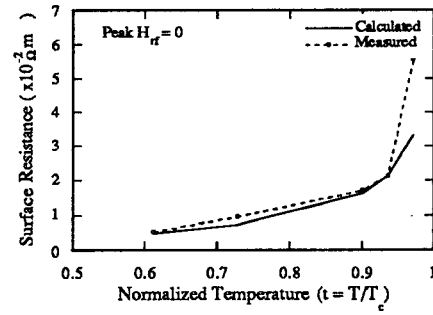


Fig. 3 Comparison between the calculated and measured [4] surface resistance for YBCO HTS as function of temperature at zero magnetic field.

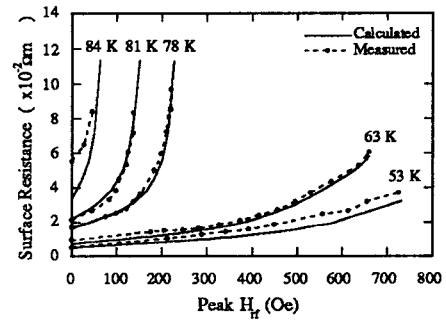


Fig. 4 Comparison between the calculated and measured [4] surface resistance for YBCO HTS as function of temperature and magnetic field.

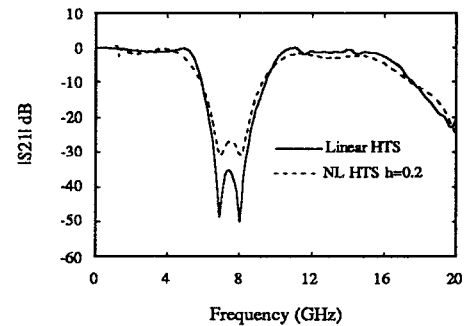


Fig. 5 Insertion loss for the low pass filter